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EFFICIENCY OF A ROUTING SCHEME IN A
MOBILE DATA PROCESSING COMMUNICATION NETWORK

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-64-635

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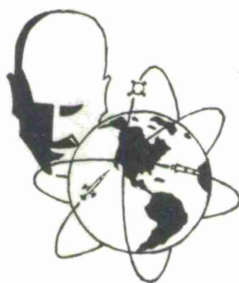
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P. Hamburger

Prepared for

DEPUTY FOR ADVANCED PLANNING
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE

L. G. Hanscom Field, Bedford, Massachusetts



Project 600.6

Prepared by

THE MITRE CORPORATION
Bedford, Massachusetts
Contract AF 19(628)-2390

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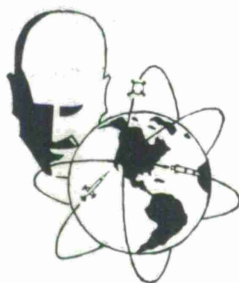
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ABSTRACT

This document examines the efficiency of a routing scheme for mobile communication nodes with data processing capability. Expressions for evaluating its cost in data rate as a function of the pertinent system variables are given. The data rate cost is compared with that of broadcasting all messages.

REVIEW AND APPROVAL

Publication of this technical documentary report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

FJ Dillon Colonel.

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EFFICIENCY OF A ROUTING SCHEME IN A MOBILE DATA PROCESSING COMMUNICATION NETWORK

INTRODUCTION

The required data rate in a communication network is a function of the knowledge available in the nodes of the network, the routing logic employed in the network, and the impressed load. It is assumed that the nodes have no prior knowledge of the positions of the users. They may acquire routing information about users which are originators of messages which they, the nodes, process. The information that they acquire is the adjacent relay to which to send a message for such users. This information may become obsolete after a time if the nodes in the network are moving and thereby changing their neighbors. If a particular user has not originated a message for a long time, the data in the nodes relating to that user is more likely to be obsolete than if that user very recently originated a message. The faster the relay nodes move with respect to one another, the quicker information stored about users is likely to become obsolete. Also, the smaller the node circle of communication, the quicker links between nodes will be broken and the quicker user information at nodes will be obsolete.

One way to circumvent the difficulty of obsolete user data in nodes is not to use it or even store it. Rather, suppose every message were sent to every relay node. That would assure the message's arrival at its destination if a path existed. The cost of such a scheme is high in data rate. Every originated message would have to be transmitted by every node. The alternative is to use the routing information unless it is found to be obsolete; when routing information is obsolete the method of sending to every relay node can still be used. Message arrival is assured just as in the more expensive scheme.

The purpose of this document is to determine the data rate advantage in using routing data acquired from messages. The reduced data rate is determined as a function of several variables. They are:

Node density per unit area

Node communication radius

Node speed

Size of network area

Message origination rate of users.

The users are assumed not to send acknowledging responses to messages. They respond only if they have information to transmit back to the sender. If users often or always respond, the following does not hold. It is also assumed that all users originate messages at the same rate on the average. In systems where quiet users are permitted, a modification to the results would be required to adequately describe the situation.

DEFINITIONS

- A = Effective area of circle (that part inside rectangle).
- A_I = Area of corners of rectangle.
- A_{II} = Area of sides of rectangle.
- A_{III} = Area of center of rectangle.
- c = Effective arc of circle, in radians, whose center is in the rectangle. That is, 2π multiplied by the fractional part of the circle circumference within the rectangle.
- L = Length of rectangle within which nodes are constrained to move.
- N = Number of nodes in rectangle.
- R = Radius of communication. Any two nodes within radius R of one another can communicate with or are audible to each other. Two nodes not within radius R of one another cannot communicate. If node N_1 can communicate with node N_2 , then the reverse is true.
- S = Constant speed at which each node is assumed to be travelling.
- S_o = Average magnitude of relative velocity of any relay with respect to any other relay.
- V_r = Relative velocity.
- W = Width of rectangle within which nodes are constrained to move.
- α = Average time between transmissions from a user. Assumed the same for all users.

- α_1 = Effective average time since the user, for whom a current message block is destined, has originated a message.
- β = Average link life.
- ϕ = Relay heading, a random variable for each node.
- ρ = Average number of relays per unit area.
- Φ = Rate of entry or exit of nodes into or from a circle of communication of a node.
- Ψ = Angle between relative velocity vector of node 1 with respect to node 2 and the radius vector from node 2 to node 1.

RESULTS

1. The magnitude of the average relative velocity of one node with respect to another is $S_r = 4S/\pi$.
2. The average number of nodes entering the circle of communication of any node per unit time is $\rho R S_r C/\pi$.
3. The average effective area/circumference ratio, A/c , depends on the section of the rectangle as shown in Table I. The sections of the rectangle are depicted in Fig. 1. Section areas as a function of L , W , and R are shown in Table I.

Table I

Area/Effective Circumference Ratio

Section*	A/c	Area (fraction of total)
A_I	$R^2/2$	$1 - [2R(L + W) + 4R^2]/LW$
A_{II}	$1.15 R^2/2$	$[2R(L + W) - 8R^2]/LW$
A_{III}	$1.2R^2/2$	$4R^2/LW$

*I, II, III are areas shown in Fig. 1.

4. The average time any node is within another node's circle of communication is the link life:

$$\beta \approx \frac{\pi^2 R}{8 S} \left[\frac{A_I + 1.15 A_{II} + 1.2 A_{III}}{A_I + A_{II} + A_{III}} \right]$$

5. Consider a node, N_0 , within the circle of communication of another node, N_1 . Independent of how long node N_0 has been in the circle, the expected value of the time interval from "now" until node N_0 exits the circle is β .

Consider nodes $N_0, N_1, \dots, N_k, \dots, N_d$ placed such that node N_k is within the circle of node N_{k+1} for $0 \leq k \leq d-1$; and a path exists between node N_0 and node N_d via nodes N_1, \dots, N_{d-1} . Then the expected time until the path is broken, i.e., the average path life, is β/d .

6. The distribution of path link lives in paths of length d , measured in links is an exponential distribution with mean β/d ; that is:

$$P_L(t, d) = \frac{d}{T} \exp(-dt/T)$$

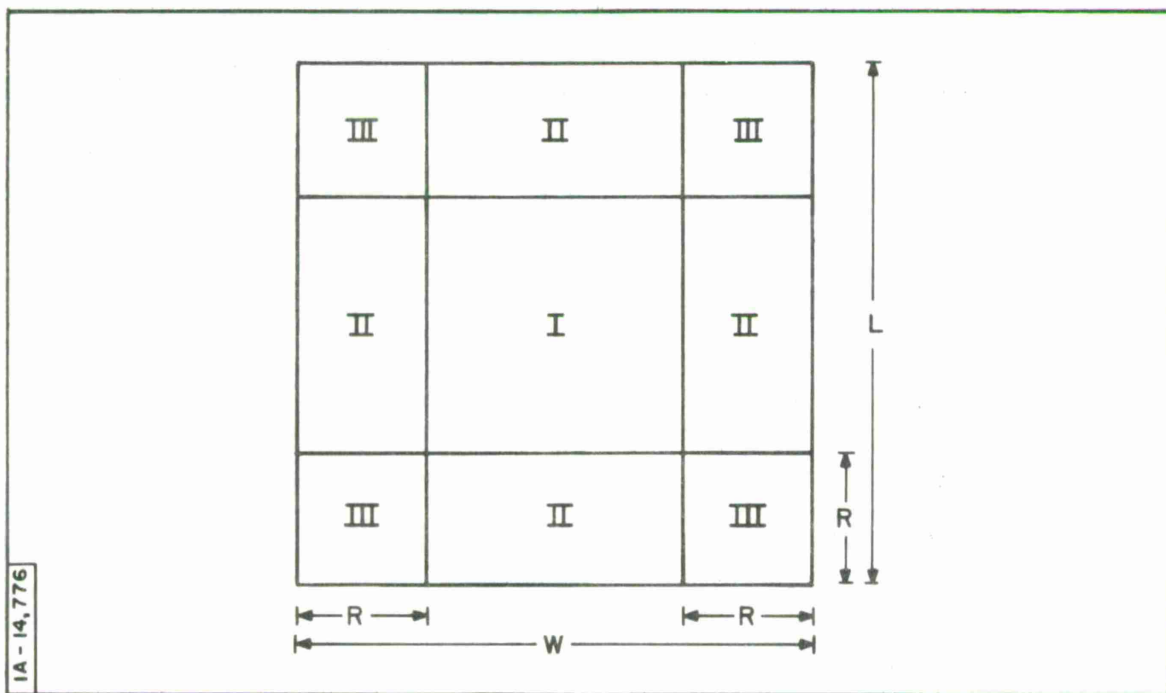


Fig. 1. Section Areas of Rectangle

7. Paths are established to a destination user whenever that user originates a message block. For the model studied, it is assumed that the distribution of waiting times between originations of any user is exponential with a mean of α ; that is $P_{\alpha}(t) = \frac{1}{\alpha} \exp(-t/\alpha)$. The distribution of ages of established paths would also be $P_{\alpha}(t)$ were it not for the following effect.
8. The ages would indeed be distributed according to $P_{\alpha}(t)$ if routing information in every message were recorded by every network node. This does occur when message blocks are transmitted in the omni mode. When they are transmitted in the addressed mode only, a fraction, f , of the nodes receive routing information. The fraction, f , is an increasing function of d , the distance between origin and receiver. The ages are distributed according to:

$$P_{\alpha_1}(t) = \frac{1}{\alpha_1} \exp(-t/\alpha_1)$$

After the network has been operating for a period at least equal to several times α , it reaches a steady state as all users have been originators for at least one message block. In this steady state, the fraction of message blocks sent addressed from origin to destination, where the most recently established path contained d links, is:

$$\frac{\beta}{\alpha_1 d + \beta}$$

9. The quantity α_1 is defined as the average time it takes for routing information to reach N nodes from one user. Its value is:

$$\alpha_1 = \frac{\alpha Nd - \beta f + \sqrt{\beta^2 f^2 + 2Nd\alpha\beta(2N - f) + \alpha^2 N^2 d^2}}{2Nd}$$

10. The final result is that the average number of node transmissions in the network per user origination is:

$$\sum_{d=1}^N \left[\left(\frac{\beta}{\alpha_1 d + \beta} \right) (d + 1) + \left(\frac{\alpha_1 d}{\alpha_1 d + \beta} \right) N \right] g(d),$$

where $g(d)$ is the probability of occurrence of paths of length, d .

This result can be compared with N , the number of node transmissions in the network per user originations.

PROOF OF RESULTS

The validity of the results of the previous section are demonstrated here. The numbers in the subsection titles correspond to the numbers of the Results statements.

1. Relative Velocity

Velocities are assumed uniform in magnitude and uniformly distributed in heading over all possible headings. From Fig. 2 and the law of cosines:

$$/V_r/{}^2 = S^2 + S^2 - 2 SS \cos \phi$$

$$= 2 S^2 (1 - \cos \phi)$$

$$/V_r/ = \sqrt{2} \quad S \sqrt{1 - \cos \phi} \quad .$$

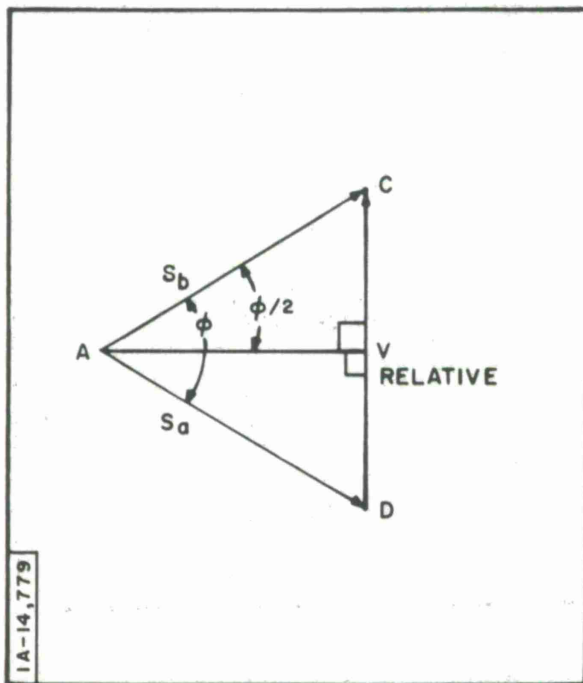


Fig. 2. Relative Velocity

Averaging over 2π radians

$$S_r = \frac{1}{2\pi} \int_0^{2\pi} \sqrt{1 - \cos \phi} d\phi = 4S/\pi .$$

2. Node Circle Entry Rate

Nodes are assumed to be uniformly distributed in space with density ρ .

The expected number of nodes in the annulus shown in Fig. 3 at any given time is $\rho c R dR$.

For the small distance, dR , it is assumed that relays travel in straight lines. Those within the distance $dR \cos \Psi$ of the inner arc will pierce the inner arc representing the radius after travelling a distance dR .

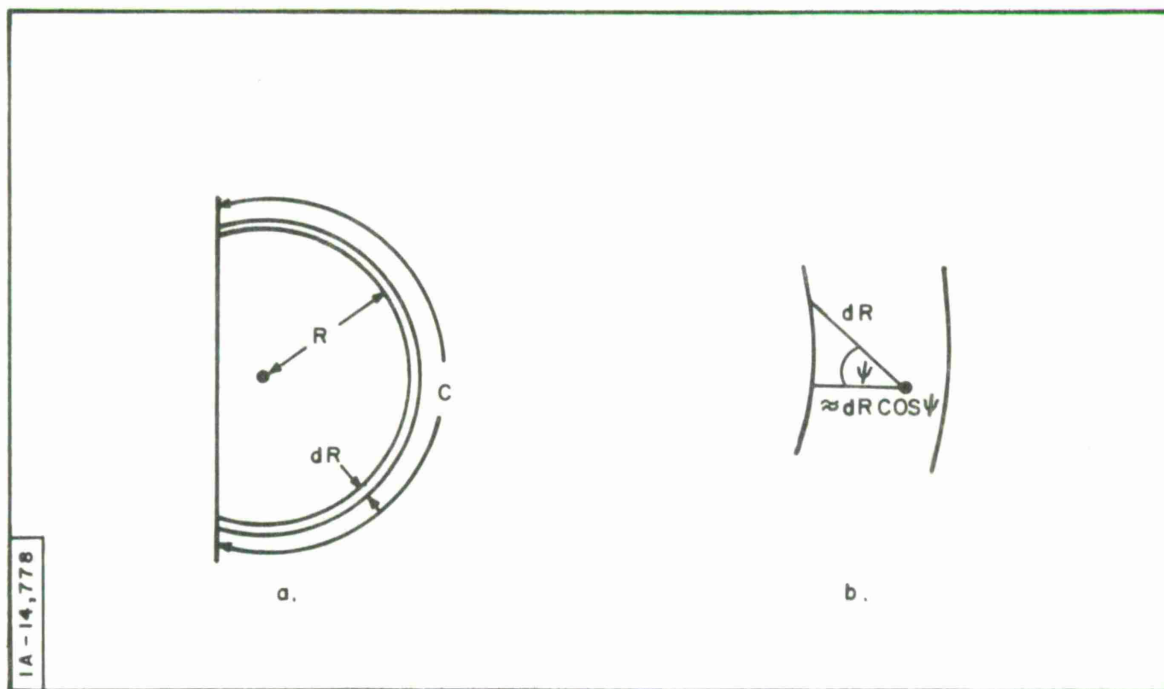


Fig. 3. Annulus

The expected value of the fraction of those relays in the annulus which will pierce the inner arc (within dR) is:

$$\frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} \cos \Psi \, d\Psi = \frac{1}{\pi} .$$

The expected value of the number entering within space dR is then:

$$\frac{\rho c R dR}{\pi} .$$

Since

$$dR = S_r \, dt ,$$

for time dt , the expected number is

$$\frac{\rho c R S_r \, dt}{\pi}$$

and for unit time:

$$\Phi = \frac{\rho c R S_r}{\pi} = \frac{4\rho c R S}{\pi^2} .$$

3. Section Areas of Rectangle

Area I, in Fig. 1, is the area in which the center of the circle of communication (a node) lies such that no part of the circle is outside the rectangle. Its area is:

$$A_I = (L - 2R) (W - 2R) = LW - 2R (W + L) + 4R^2 .$$

Area II is the area in which the center of the circle of communication (a node) lies such that part of the circle lies outside the rectangle but only on one side.

Its area is:

$$\begin{aligned} A_{II} &= 2(L - 2R) R + 2 (W - 2R) R \\ &= 2R (L + W - 4R) \\ &= 2R (L + W) - 8R^2 . \end{aligned}$$

Area III is the area in which the center of the circle of communication (a node) lies such that part of the circle lies outside the rectangle on two sides. Its area is:

$$A_{III} = 4R^2 .$$

4. Average Time within Circle of Communication

Consider the circle of communication of one node. The average rate of entry (or exit) of other nodes for that circle is Φ and has been derived in subsection 2, above. If all nodes in the circle, except the one defining its center, are assumed to have entered the circle previously and are assumed to exit the circle later, one can say:

$$(\text{Average Entry rate}) \times (\text{Average Duration of stay in circle}) =$$

$$(\text{Average number of nodes in circle}^*)$$

or

$$\Phi \beta = \rho A .$$

* Discounting the center node.

Rearranging we have,

$$\beta = \frac{\rho A}{\Phi} .$$

It must be pointed out that Φ contains ρ as a factor. Therefore, ρ is not really a random variable in the right side of the equation and need not be taken into account in the averaging process which follows. As shown by the substitution in the next equation, the only random variables contributing to Φ are A , c , and S_r which is expressed as $4S/\pi$.

Consider the random variables S_r and A/c . These two are statistically independent. The average of S_r is computed in subsection 1 and the average of A/c is computed later in this section. The resulting averages are directly divided. The variables A and c are not statistically independent. Indeed, they both vary according to which area of Fig. 1 the node happens to be in. Therefore the mean of A/c must be computed.

The quantity Φ is given in subsection 2. Substituting in the last equation produces:

$$\beta = \frac{\pi^2}{4RS} \left(\frac{A}{c} \right) ,$$

The quantity A/c must be evaluated separately for each of the three sections of the rectangle of Fig. 1.

Section I (See Fig. 4a)

$$A = \pi R^2$$

$$c = 2\pi$$

$$A/c = R^2/2$$

$$\beta = \pi^2 R/8S$$

Section II (Fig. 4b)

Let x = fractional part of the radius between node and rectangle side.

From the figure:

$$A = \frac{R^2}{2} \left(c + x \sqrt{1 - x^2} \right) ,$$

$$c = 2 \left(\pi - \cos^{-1} x \right) .$$

Then,

$$\left(\frac{A}{c} \right) = \frac{R^2}{2} \left[1 + \frac{x \sqrt{1 - x^2}}{\pi - \cos^{-1} x} \right] .$$

Averaging over the possible range of x ,

$$\beta = \frac{\pi^2 R}{8 S} \frac{\int_0^1 \left(1 + \frac{x \sqrt{1 - x^2}}{\pi - \cos^{-1} x} \right) dx}{\int_0^1 dx} ,$$

$$\beta = \frac{\pi^2 R}{8 S} \left[1 + \int_0^1 \frac{x \sqrt{1 - x^2}}{\pi - \cos^{-1} x} dx \right] .$$

The integral was approximately evaluated using 11 points and Simpson's rule. The result produced

$$\tau = \frac{\pi^2 R}{8 S} [1.151] .$$

Section III (Figs. 4c and 4d)

Let x = the fractional part of the radius between the node and the vertical edge of the rectangle, and let y = the fractional part of the radius between the node and the horizontal edge of the rectangle. Further subdivide section III into subsections IIIa and IIIb; IIIa is the subsection with one radius of a rectangle corner. Section IIIb is the remainder of section III.

Consider section IIIa:

$$A = \frac{R^2}{2} (c + xy) + \frac{1}{2} x \sqrt{1 - x^2} + \frac{1}{2} y \sqrt{1 - y^2} ,$$

$$c = \frac{\pi}{2} + \sin^{-1} x + \sin^{-1} y .$$

Now in section IIIb:

$$A = \frac{R^2}{2} (c) + x \sqrt{1 - x^2} + y \sqrt{1 - y^2} ,$$

$$c = \frac{\pi}{2} + \sin^{-1} x + \sin^{-1} y + \sin^{-1} y - \cos^{-1} x ,$$

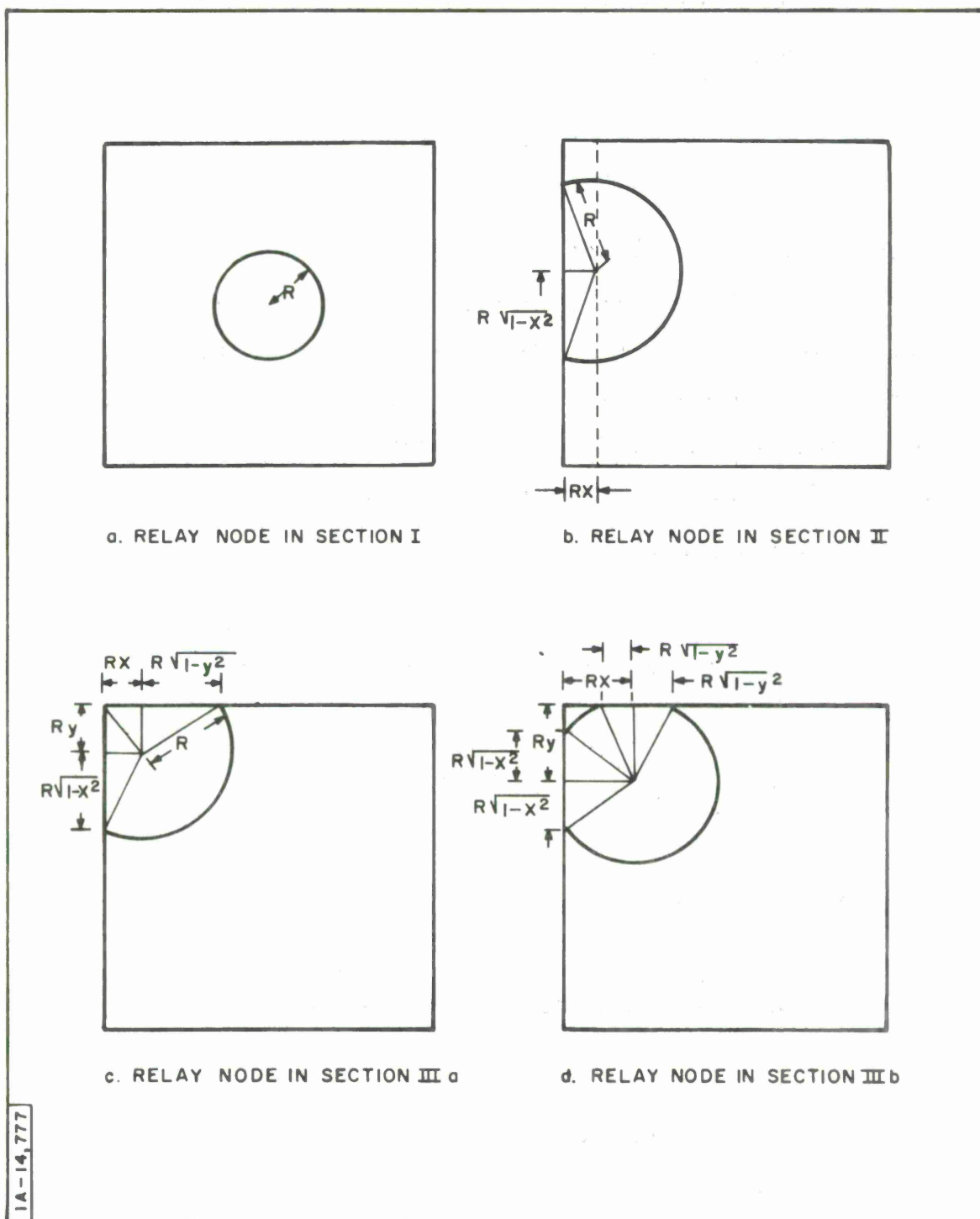


Fig. 4. Node Location

gives:

$$c = 2 \sin^{-1} x + 2 \sin^{-1} y .$$

Substituting:

$$\beta = \frac{\pi^2 R}{8 S} \left\{ \int_0^1 \int_0^{\sqrt{1-y^2}} \left[1 + \frac{xy + \frac{1}{2} x \sqrt{1-x^2} + \frac{1}{2} y \sqrt{1-y^2}}{\frac{\pi}{2} + \sin^{-1} x + \sin^{-1} y} \right] dx dy \right. \\ \left. + \int_0^1 \int_{\sqrt{1-y^2}}^1 \left[1 + \frac{x \sqrt{1-x^2} + y \sqrt{1-y^2}}{2 \sin^{-1} x + 2 \sin^{-1} y} \right] dx dy \right\} \div \int_0^1 \int_0^1 dx dy .$$

The substitutions,

$$x = \sin w \quad dx = \cos w \, dw$$

$$y = \sin z \quad dy = \cos z \, dz ,$$

were made in order to make the integrand easier to evaluate using a FORTRAN STRETCH computer program.

Substituting the last equation into the previous one, we get

$$\tau = \frac{\pi^2 R}{8 S} \times \left\{ 1 + \int_0^{\pi/2} \int_0^{\cos z} \frac{\sin w \sin z + \frac{1}{2} \sin w \cos w + \frac{1}{2} \sin z \cos z}{\frac{\pi}{2} + w + z} \cos w \cos z \, dw \, dz \right. \\ \left. + \int_0^{\pi/2} \int_{\cos z}^{\pi/2} \frac{\sin w \cos w + \sin z \cos z}{2 (w + z)} \cos w \cos z \, dw \, dz \right\}.$$

This equation was integrated by summing the values at 10,000 points equally spaced over the space of integration and multiplying the result by 1/10,000 the area of the w-z plane under the surface of the volume being integrated. The first and second integrals were evaluated as .1597 and .055 respectively. Thus, substituting numerical values gives the result indicated in subsection 4 of the Results.

5. Path Life Is Inversely Proportional to Path Length, d

Consider separately each link in the path. On the average a single link will break $1/\beta$ times per unit time where β is the average life. All d links will break d/β times per unit time on the average. Therefore the average life of the d links making up the path is β/d .

6. Distribution of Path Lives

An exponential distribution of path lives is postulated. Path life terminations are events which can occur with equal likelihood at any time. Waiting

times for such events in continuous time have been shown by Feller^[1] and others to be exponentially distributed.

7. Distribution of Intervals Between Path Creation

It is assumed that any user may desire to originate a message with equal probability at any time. Again, application of Feller's work leads us to the exponential distribution of time intervals between creation of paths originating with any user. The age of a path is reckoned from the time the most recent message was originated until "now," regardless of whether its "life" has terminated. Thus, if a mean time between origins at a particular node is α , the times between origins are distributed according to $P_{\alpha}(t) = \frac{1}{\alpha} \exp(-t/\alpha)$. Also, the age of the set of all paths to all nodes is distributed according to the same $P_{\alpha}(t)$. However paths established by addressed messages may not reach all relays but only a fraction, $f(d)$, of the relays. Subsequent message blocks originating from a user communicating with a relay within the fraction, $f(d)$, will take advantage of the newly established path. If the corresponding relay is outside that fraction of relays, the message will not take advantage of the newly established path. The effective mean path age resulting is next derived.

8. Mean Probability of Path Survival

Given that the ages of paths are distributed according to

$$\frac{1}{\alpha_1} \exp(-t/\alpha_1) ,$$

[1] Feller, W. An Introduction to Probability Theory and its Applications, Wiley, 1960, pp. 411-412.

and that the lives of the paths are distributed according to

$$\frac{d}{\beta} \exp(-dt/\beta) ,$$

it is asserted that the probability that a path is established, and addressed messages are sent, when a directed message block is transmitted as:

$$P(\text{addressed}) = \int_0^{\infty} \left(\int_{\tau}^{\infty} \frac{1}{b} \exp(-t/b) dt \right) \left(\frac{1}{a} \exp(-\tau/a) \right) d\tau ,$$

where $b = \beta/d$; $a = \alpha_1$.

This equation expresses the mean probability of path survival, which is the probability of survival for age t weighted with the probability of occurrence of age t for a path averaged for all ages from 0 to ∞ .

Evaluation produces

$$\begin{aligned} P(\text{addressed}) &= \int_0^{\infty} \left(\int_{\tau}^{\infty} \exp(-t/b) dt \right) \frac{1}{a} \exp(-\tau/a) d\tau \\ &= \int_0^{\infty} \frac{1}{a} \exp(-\tau/b) \exp(-\tau/a) d\tau \\ &= \frac{1}{a} \int_0^{\infty} \exp(-\tau(a+b)/ab) d\tau \end{aligned}$$

$$= \frac{-b}{a+b} \exp(-t(a+b)/ab) \Big|_0^{\infty}$$

$$= \frac{b}{a+b} .$$

Reversing substitutions made above yields:

$$P(\text{addressed}) = \frac{\beta}{\alpha_1 d + \beta} ,$$

$$P(\text{omni}) = 1 - P(\text{addressed}) ,$$

$$P(\text{omni}) = \frac{\alpha_1 d}{\alpha_1 d + \beta} .$$

9. Derivation of Effective Average Age, α_1

Subsection 8 assumed an effective average age, α_1 .

Definition: α_1 = the time a particular node must wait between message blocks from a particular user averaged for all users, all nodes, and all time.

If all message blocks are sent to all nodes, i.e., omni node of transmission (this would occur if α , average time between user originations, were very large compared to β , path life) then:

$$\alpha = \alpha_1 .$$

If all messages were sent addressed, then f nodes would gain routing information, and it would take N/f message blocks to spread the same amount of routing information to the nodes of the network as could be spread by one

message block propagated throughout the network by omni message blocks. This would occur if β , path life, were very large compared to α , average time between user originations (e.g., a static network. Then:

$$\alpha_1 = \alpha N / f .$$

The quantity, $1/\alpha_1$, is interpreted to mean the average number of nodes receiving routing information about one user per unit time divided by the number of nodes in the system. Count a single node receiving information twice as two. Correspondingly, $1/\alpha$ is interpreted as the average number of messages originated per user per unit time. From subsection 8, we have the number of message blocks sent addressed and omni. By definition, an addressed message block sends routing data to f nodes and an omni message block sends routing data to N nodes.

Hence,

$$\frac{1}{\alpha_1} = \frac{1}{\alpha} \left[\frac{\frac{\beta}{\alpha_1 d + \beta} f + \frac{\alpha_1 d}{\alpha_1 d + \beta}}{N} \right] .$$

This gives rise to a quadratic equation in α_1 :

$$dN \alpha_1^2 + (\beta f - \alpha N d) \alpha_1 - N \alpha \beta = 0 .$$

The solutions to this equation are:

$$\alpha_1 = \frac{\alpha N d - \beta f \pm \sqrt{\beta^2 f^2 + 2 N d \alpha \beta (2N-f) + \alpha^2 N^2 d^2}}{2Nd} .$$

This result can be shown to equal the values of $\alpha = \alpha_1$ and $\alpha_1 = \alpha N/f$ for the limiting values of α/β as follows:

Pull β from the result

$$\alpha_1 = \frac{\beta \frac{\alpha}{\beta} Nd - f + \sqrt{f^2 - 2fNd \frac{\alpha}{\beta} + 4N^2 d \frac{\alpha}{\beta} + N^2 d^2 - \frac{\alpha^2}{\beta^2}}}{2Nd};$$

eliminate second order infinitesimal and combine terms

$$\lim_{\frac{\alpha}{\beta} \rightarrow 0} \alpha_1 = \frac{\alpha Nd - f\beta + \beta \sqrt{f^2 + 2Nd(2N-f) \frac{\alpha}{\beta}}}{2Nd};$$

and

$$\lim_{\frac{\alpha}{\beta} \rightarrow 0} \alpha_1 = \frac{\alpha Nd - f\beta + \beta \sqrt{f^2 + \frac{Nd(2N-f)}{f} \frac{\alpha}{\beta}}}{2Nd};$$

approximate the square root

$$\lim_{\frac{\alpha}{\beta} \rightarrow 0} \alpha_1 = \frac{\alpha Nd - f\beta + \beta f + \frac{\beta Nd(2N-f)}{f} \frac{\alpha}{\beta}}{2Nd};$$

rationalize numerator

$$\lim_{\frac{\alpha}{\beta} \rightarrow 0} \alpha_1 = \frac{\alpha N d f + \beta N d (2N - f) \alpha / \beta}{2 N d f} ;$$

combine the terms

$$\lim_{\frac{\alpha}{\beta} \rightarrow 0} \alpha_1 = \frac{\alpha f + \alpha (2N - f)}{2f} ;$$

and finally

$$\lim_{\frac{\alpha}{\beta} \rightarrow 0} \alpha_1 = \frac{\alpha N}{f} .$$

Now pull α from the result

$$\alpha_1 = \alpha \left[\frac{N d - \frac{\beta}{\alpha} f + \sqrt{\frac{\beta^2}{\alpha^2} f^2 - 2 \frac{\beta}{\alpha} f N d + 4 \frac{\beta}{\alpha} N^2 d + N^2 d^2}}{2 N d} \right] .$$

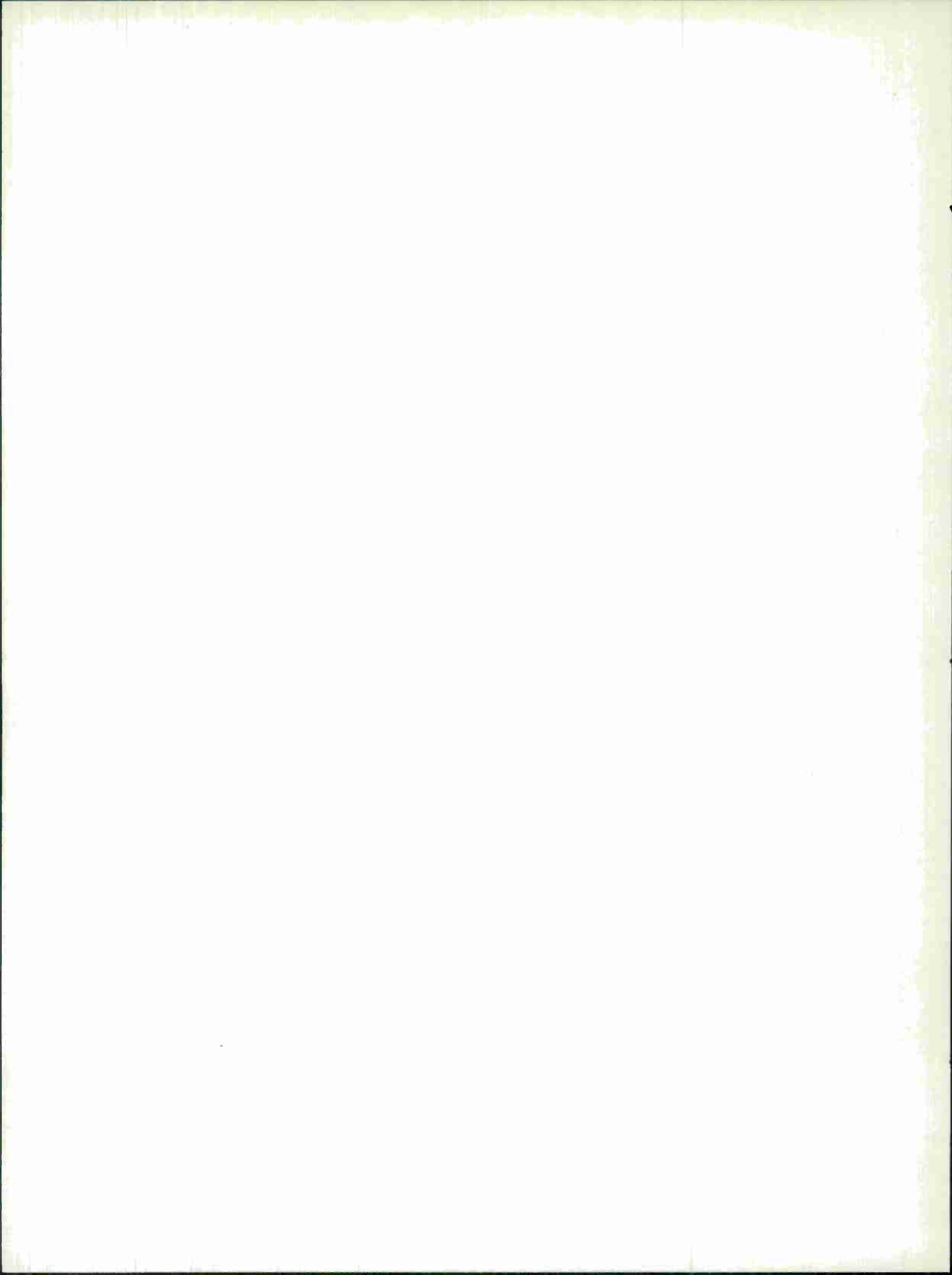
Proceed in the same manner as above:

$$\lim_{\frac{\beta}{\alpha} \rightarrow 0} \alpha_1 = \alpha \left[\frac{Nd - \frac{\beta}{\alpha} f + \sqrt{\left(Nd + \frac{\beta}{\alpha} [2N - f]\right)^2}}{2 Nd} \right]$$

$$= \left[\frac{Nd - \frac{\beta}{\alpha} f + Nd + \frac{\beta}{\alpha} (2N - f)}{2 Nd} \right] ;$$

$$\lim_{\frac{\beta}{\alpha} \rightarrow 0} \alpha_1 = \alpha \frac{2Nd}{2Nd} = \alpha .$$


P. Hamburger



APPENDIX

EXAMPLE

To illustrate the results, an example is presented. The parameters chosen roughly represent a network of subsonic aircraft over the continental U. S. Assume the following parameters:

α (average time between
originations of each user) = 10 minutes = $\frac{1}{6}$ hour

N (number of nodes) = 60

R (communication radius) = 300 miles

L (length of total area) = 3000 miles

W (width of total area) = 1500 miles

f (number of nodes receiving routing data from
Addressed Messages) = $3d + 3$, where

d = distance in links from origin to destination.

g (d), the frequency of message transmissions as a function
of link distance to be travelled, is given by Table II.

Table II

d	1	2	3	4	5	6	7
g(d)	0.30	0.25	0.20	0.12	0.08	0.04	0.01

Step 1: The fractional areas are calculated from Table I of the text.

$$\frac{A_I}{A_I + A_{II} + A_{III}} = \frac{126}{225}$$

$$\frac{A_{II}}{A_I + A_{II} + A_{III}} = \frac{63}{225}$$

$$\frac{A_{III}}{A_I + A_{II} + A_{III}} = \frac{36}{225}$$

Step 2: The average link life, β , is calculated by the equation in subsection 4 under Results.

$$\beta = .6625 \text{ hours}$$

Step 3: The effective average time between receipt of routing information about each user by a node, α_1 , is a function of distance.

Table III gives α_1 in hours as a function of d , as computed by the equation of subsection 9 under Results.

Table III

d	1	2	3	4	5	6	7
α_1	0.387	0.333	0.263	0.240	0.225	0.213	0.208

(Note: α_1 tends toward $\alpha = .167$ as the distance increases, $d \rightarrow N_1$, $f(d) \rightarrow N$, and all nodes receive all messages. If that is the case, $\alpha_1 = \alpha$)

Step 4: The equation of subsection 10 of the Results is now applied to get the number of transmissions in the network per user origination. The coefficient of $g(d)$ for each value of d is listed in Table IV.

Table IV

d	1	2	3	4	5	6	7
n	23	31	33	36	38	40	42

n = number of transmissions per message in the network
using Addressing.

The numbers in Table IV can now be weighted by Table II and summed to get an average as suggested in subsection 10 of the Results.

The average number of transmissions in the network per user origination is:

30.7

This compares with 60 for use of broadcast messages and no routing. Thus, just over half the transmission bit rate for information transfer is required.

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13. ABSTRACT This document examines the efficiency of a routing scheme for mobile communication nodes with data processing capability. Expressions for evaluating its cost in data as a function of the pertinent system variables are given. The data rate cost is compared with that of broadcasting all messages.			

14	KEY WORDS	LINK A		LINK B		LINK C	
		ROLE	WT	ROLE	WT	ROLE	WT
Communication Networks Data Processing, Mobile Data Rate Data Processing Systems							

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